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DYNAMIC STRUCTURAL MODEL OF A SUBMERGED RING.(U)
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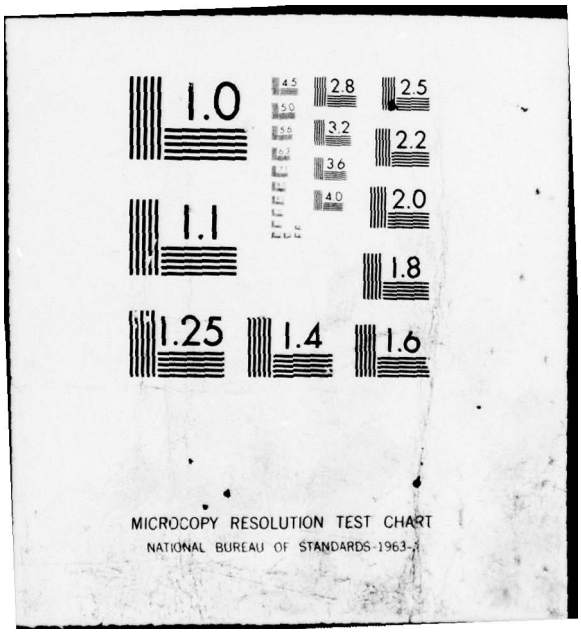
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THESIS

DYNAMIC STRUCTURAL MODEL
OF A SUBMERGED RING

by

Jack Thomas Waller Jr.

September 1979

Thesis Advisor: R. E. Newton

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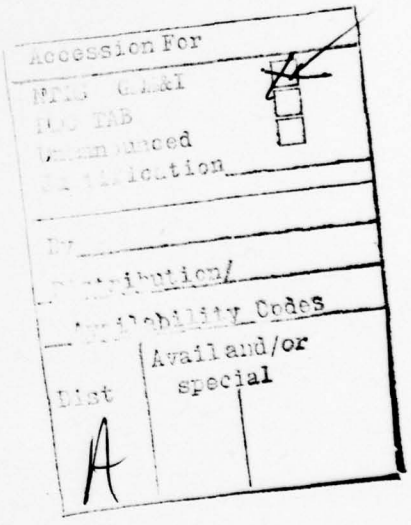

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Dynamic Structural Model of a Submerged Ring

by

Jack Thomas Waller Jr.
B.S.M.E., New Mexico State University, 1975

Submitted in partial fulfillment of the requirements for the degree of

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ABSTRACT

A dynamic structural model of a submerged ring is developed using trigonometric series. It is constructed for use in conjunction with a finite element fluid model to examine the effects of cavitation on underwater shock loading of a structure. The governing equations and the time integration algorithm used in the model are described. Results predicted by the model are compared to known results. The program listing is given.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS

A	-	cross sectional area of the ring (m^2)
a_n	-	Fourier cosine coefficient for radial displacement (m)
b_n	-	Fourier sine coefficient for tangential displacement (m)
c	-	speed of sound in ring material (m/s)
c_n	-	Fourier cosine coefficient for normal pressure (Pa)
D	-	flexural rigidity of the ring ($N \cdot m^2$)
E	-	modulus of elasticity of the ring material (Pa)
h	-	time step used in central difference integration (sec)
I	-	area moment of inertia about centroidal axis (m^4)
κ	-	curvature of the ring (1/m)
M	-	bending moment about centroidal axis ($N \cdot m$)
ω_n	-	natural circular frequency of vibration (rad/s)
r	-	centroidal radius of gyration of ring cross section (m)
R	-	radius of ring (m)
ρ	-	density of ring material (kg/m^3)
θ	-	angle between normal to shock plane and location on the ring (radians)
v	-	tangential displacement (m)
w	-	radial displacement (m)
z	-	$(r/R)^2$

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I. INTRODUCTION

The program evolved during this study models a submerged circular ring using trigonometric series. It was developed for use in conjunction with a finite element fluid model based on a displacement potential formulation. The purpose of the combined models is to predict the effects of cavitation on underwater shock loading of the structure. More information on the fluid model can be found in References 1 and 2. The purpose of this paper is to describe the development of the structural model and to present some of the results obtained from its use in combination with the fluid model. A listing of the FORTRAN IV program implementing the structural model is given in the Appendix.

II. GOVERNING EQUATIONS

A. DIFFERENTIAL EQUATIONS FOR THE DEFLECTION OF A THIN CIRCULAR RING

In Figure 1 the solid line represents the ring after deformation, and the dotted line the ring before deformation. For small deflections the curvature of the element $m_1 n_1$ can be taken as

$$\frac{1}{R_1} = \frac{d\theta + \Delta d\theta}{ds + \Delta ds}$$

where R, w, θ and s are defined as shown in Figure 1 and w is taken positive inward. Making use of the relations

$$\Delta d\theta = d\theta_1 - d\theta = \frac{dw}{ds} + \frac{d^2w}{ds^2} ds - \frac{dw}{ds} = \frac{d^2w}{ds^2} ds$$

$$\Delta ds = ds_1 - ds = (r - w)d\theta - rd\theta = -wd\theta = -w \frac{ds}{R}$$

and substituting into the equation above yields

$$\frac{1}{R_1} = \frac{d\theta + \frac{d^2w}{ds^2} ds}{ds(1 - \frac{w}{R})} = \frac{d\theta(1 + \frac{w}{R})}{ds(1 - \frac{w}{R})(1 + \frac{w}{R})} + \frac{\frac{d^2w}{ds^2} ds(1 + \frac{w}{R})}{ds(1 - \frac{w}{R})(1 + \frac{w}{R})}$$

Neglecting the higher order terms, the above expression becomes

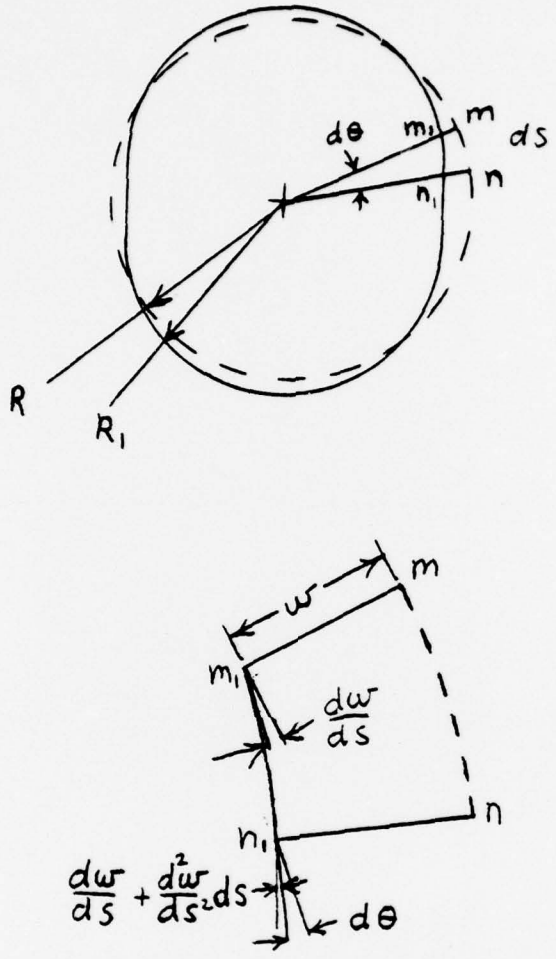

Figure 1. Geometry of Ring Deformation

$$\frac{1}{R_1} = \frac{d\theta}{ds}\left(1 + \frac{w}{R}\right) + \frac{d^2w}{ds^2} = \frac{1}{R}\left(1 + \frac{w}{R}\right) + \frac{d^2w}{ds^2}$$

or alternately

$$\frac{1}{R_1} - \frac{1}{R} = \frac{w}{R^2} + \frac{d^2w}{ds^2} = \frac{1}{R^2}\left(w + \frac{d^2w}{d\theta^2}\right) \qquad (1)$$

For a ring where the thickness is small compared to the radius and elastic behavior is assumed, it can be shown that the approximate relationship between deflection and loading is [Reference 7]

$$\frac{1}{R_1} - \frac{1}{R} = -\frac{M}{D} \qquad (2)$$

where M is the bending moment about the centroidal axis and D is the flexural rigidity of the ring. A positive bending moment produces compression in the outside fibers of the ring. Combining equations 1 and 2 yields the differential equations for the deflection of the ring given below.

$$\frac{1}{R^2}\left(\frac{\partial^2 w}{\partial\theta^2} + w\right) = -\frac{M}{D} \qquad (3)$$

B. GOVERNING EQUATIONS OF MOTION

The governing equations of motion used in the program were arrived at by the application of Hamilton's principle. In order to apply Hamilton's principle, it is first necessary to derive expressions for the strain energy and kinetic energy of the ring as well as the work done by the external loads. In these derivations the ring was taken to be homogeneous, elastic, and of unit width. The pressures on the ring and its deflection were represented by the pressures at and the deflection of a set of nodal points equally spaced along the circumference of the ring.

The shock front is assumed to approach the ring normal to the $\theta = 0$ plane as shown in Figure 2, where θ is taken to be positive counterclockwise.

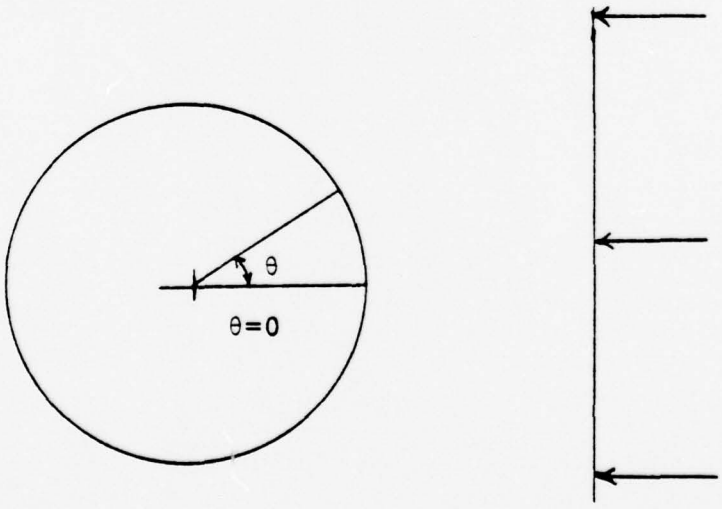

Figure 2. Ring, Shock Front Orientation

Since the ring is symmetric about the $\theta = 0$ plane, results are given for θ from 0° to 180°. The response of the ring is represented by the response of a set of N+1 nodal points equally spaced along the ring. The radial displacement is approximated by a trigonometric cosine series

$$w = \sum_{n=0}^{N} a_n \cos n\theta \quad . \tag{4}$$

The tangential deflection is represented by the trigonometric sine series

$$v = \sum_{n=1}^{N} b_n \sin n\theta \quad , \tag{5}$$

where v is taken to be positive in the negative θ direction. Similarly the normal pressure applied to the ring is represented by

$$p = \sum_{n=0}^{N} c_n \cos n\theta \quad . \tag{6}$$

1. Strain Energy

The strain energy is comprised of two components [Ref. 4]. One component is the strain energy due to bending and the other to strain in the θ direction (ε_θ). The strain energy stored in the ring due to bending is calculated as

the work done by the internal moment acting over the entire ring, and for a ring of unit width is

$$U_B = \frac{1}{2} \int_0^{2\pi} M R\kappa d\theta$$

where κ is the curvature of the ring. Combining this equation with equation 3 and using the relation $\kappa = M/D$ yields

$$U_B = \frac{1}{2} \int_0^{2\pi} D \frac{1}{R^3} \left(\frac{\partial^2 w}{\partial \theta^2} + w\right)^2 d\theta \qquad (7)$$

The strain energy due to the strain in the θ direction is obtained as the distance traveled by the average normal force in the θ direction.

The circumferential strain is composed of two components the first of which is the result of the radial displacement of the ring ($\frac{w}{R}$) and the second results from tangential displacement and is ($\frac{1}{R} \frac{\partial v}{\partial \theta}$). The strain energy from the normal force (N) is

$$U_\theta = \frac{1}{2} \int_0^{2\pi} N\varepsilon_\theta R \, d\theta = \frac{1}{2} \int_0^{2\pi} A\varepsilon_\theta E\varepsilon_\theta R \, d\theta$$

or (8)

$$U_\theta = \frac{1}{2} \int_0^{2\pi} RAE\varepsilon_\theta^2 \, d\theta = \frac{1}{2} \int_0^{2\pi} \frac{AE}{R^2} \left(\frac{\partial v}{\partial \theta} + w\right)^2 Rd\theta$$

where A is the cross sectional area of the ring and

E is the modulus of elasticity of the ring material.

Combining equations 7 and 8, the expression for the total strain energy is

$$U_T = \frac{1}{2} \int_0^{2\pi} \frac{D}{R^3} \left(\frac{\partial^2 w}{\partial \theta^2} + w\right)^2 d\theta + \frac{1}{2} \int_0^{2\pi} \frac{AE}{R} \left(\frac{\partial v}{\partial \theta} + w\right)^2 d\theta \qquad (9)$$

Substituting into this equation the series expressions for radial and tangential displacements, and assuming constant geometrical and material properties over the ring yields after integration

$$U_T = \frac{\pi D}{2R^3} \left[\sum_{n=0}^{N} a_n^2 (n^2 - 1)^2 + \frac{AE\pi}{2R} \left[\sum_{n=0}^{N} (nb_n + a_n)^2 \right] \right] \quad *$$

An expression for the change in strain energy δU corresponding to small changes in tangential and radial displacements (δb_n and δa_n) can now be found

$$\delta U = \frac{\pi D}{3} \left[\sum_{n=0}^{N} (n^2 - 1)^2 a_n \delta a_n \right] + \frac{AE\pi}{R} \left[\sum_{n=0}^{N} (n^2 b_n + n a_n) \delta b_n + (nb_n + a_n) \delta a_n) \right] \qquad (10)$$

---

*In this equation and several that follow which contain a summation expression starting with n = 0, a factor of 2 associated with the n = 0 term is omitted for the sake of brevity. The term is accounted for in the solution process used with the model.

2. Work

The work associated with a small change in radial displacement is given by

$$\delta W = \int_0^{2\pi} p \, \delta w \, R \, d\theta$$

Substituting the relations

$$p = \sum_{m=0}^{N} c_m \cos m\theta$$

and

$$\delta w = \sum_{n=0}^{N} \delta a_n \cos n\theta$$

into this expression yields, after integration

$$\delta W = \sum_{n=0}^{N} c_n \, \delta a_n \, \pi R$$

3. Kinetic Energy

The expression for the kinetic energy of the ring can be arrived at by considering an infinitesimal section of the ring that has been set in motion. The kinetic energy of the element can be represented as the sum of a contribution due to translation of the mass center and a contribution due to rotation. This leads to

$$dT = \frac{1}{2}\rho(ARd\theta)(\dot{w}^2 + \dot{v}^2) + \frac{1}{2}\rho IRd\theta\left(\frac{\partial \dot{w}}{R\partial\theta}\right)^2$$

where I is the area moment of inertia about the centroidal axis.

Using the series relations

$$w = \sum_{n=0}^{N} a_n \cos n\theta \qquad \dot{w} = \sum_{n=0}^{N} \dot{a}_n \cos n\theta$$

$$v = \sum_{n=1}^{N} b_n \sin n\theta \qquad \dot{v} = \sum_{n=1}^{N} \dot{b}_n \sin n\theta$$

yields

$$dT = \frac{1}{2}\rho(ARd\theta)\left[\left(\sum_{n=0}^{N} \dot{a}_n \cos n\theta\right)^2 + \left(\sum_{n=1}^{N} \dot{b}_n \sin n\theta\right)^2\right]$$

$$+ \frac{1}{2}\rho IRd\theta\left(-\frac{1}{R}\sum_{n=1}^{N} \dot{a}_n\, n \sin n\theta\right)^2$$

Integration over the ring yields

$$T = \frac{\pi\rho}{2}\sum_{n=0}^{N}\left[\left(AR + n^2\frac{I}{R}\right)\dot{a}_n^2 + AR\dot{b}_n^2\right]$$

The resultant change in kinetic energy due to a small change in velocity components can then be found as

$$\delta T = \pi\rho\sum_{n=0}^{N}\left[\left(AR + n^2\frac{I}{R}\right)\dot{a}_n\delta\dot{a}_n + AR\dot{b}_n\delta\dot{b}_n\right] \qquad (12)$$

4. Application of Hamilton's Principle To Find Coupled Equations of Motion

From Hamilton's principle [Ref. 5] it is known that

$$\int_{t_1}^{t_2} (\delta T - \delta U + \delta W)\, dt = 0 \qquad (13)$$

Combining equations 10, 11, 12 and 13 yields, after carrying out an integration by parts on the first term,

$$\sum_{n=0}^{N} \pi\rho\{AR + n^2 \frac{I}{R})[\dot{a}_n \delta a_n \Big|_{t_1}^{t_2} - \int_{t_1}^{t_2} \ddot{a}_n\, \delta a_n\, dt]$$

$$+ \sum_{n=0}^{N} \pi\rho AR[\dot{b}_n\, \delta b_n \Big|_{t_1}^{t_2} - \int_{t_1}^{t_2} \ddot{b}_n\, \delta b_n\, dt]$$

$$- \int_{t_1}^{t_2} \{\frac{\pi D}{R^3}[\sum_{n=0}^{N} (n^2-1)^2\, a_n\, \delta a_n] + \frac{AE\pi}{R}[\sum_{n=0}^{N} (n^2 b_n + n a_n)\delta b_n$$

$$+ (n b_n + a_n)\delta a_n] - \sum_{n=0}^{N} c_n\, \delta a_n\, \pi R\}\, dt = 0$$

Since the δa_n and δb_n can be chosen arbitrarily, this becomes

$$(1 + (\frac{r}{R})^2 n^2)\ddot{a}_n + (\frac{c}{R})^2[1 + (\frac{r}{R})^2(n^2 - 1)^2]a_n + (\frac{c}{R})^2 = \frac{c_n}{\rho A}$$

$$\ddot{b}_n + (\frac{c}{R})^2 n a_n + (\frac{c}{R})^2 n^2 b_n = 0 \qquad (14)$$

where $c = (E/\rho)^{1/2}$ is the speed of sound in the ring and

where r is the centroidal radius of gyration of the ring section and the relation $I = Ar^2$ has been used.

5. Separated Equations of Motion

It is common practice to separate ring deformations into flexural modes and extensional modes. The initial version of this program utilized such a resolution. The flexural mode is characterized by the requirement that

$$\varepsilon_\theta = -(w + \frac{\partial v}{\partial \theta})/R = 0 .$$

This implies that the Fourier coefficients satisfy the relation

$$a_{n_{(F)}} + nb_{n_{(F)}} = 0 \qquad (15)$$

The extensional mode is defined to be geometrically orthogonal to the flexural mode so that

$$na_{n_{(E)}} - b_{n_{(E)}} = 0 \qquad (16)$$

Substituting equation 15 into equations 14 yields the result

$$[\rho A((\frac{n^2+1}{n^2}) + zn^2)]\ddot{a}_{n_{(F)}} + [\frac{EA}{R^2} z(n^2-1)^2]\, a_{n_{(F)}} = c_n$$

$$b_{n_{(F)}} = -a_{n_{(F)}}/n \qquad (17)$$

where $z = (r/R)^2$.

If equation 16 is substituted into equations 14, the result is

$$[\rho A((n^2+1) + zn^2)]\ddot{a}_{n_{(E)}} + [\frac{EA}{R^2}((n^2+1)^2 + z(n^2-1)^2]a_{n_{(E)}} = c_n \qquad (18)$$

$$b_{n_{(E)}} = na_{n_{(E)}}$$

Initial solutions were carried out using equations 17 and 18 and then combining the results by

$$a_n = a_{n_{(F)}} + a_{n_{(E)}}$$

$$b_n = b_{n_{(F)}} + b_{n_{(E)}}$$

This procedure gave results which satisfactorily predicted the ring bending moments but gave poor accuracy for axial force determinations.

Re-examination of the foregoing solution process revealed that the mode shapes defined by equations 15 and 16 are not orthogonal with respect to the mass or the stiffness matrices of the system. It was accordingly decided to return to equations 14 and solve them

simultaneously in the time integration algorithm. This change led to accurate axial force determinations. The separated equations of motion, 17 and 18, are used in the program as described in Section III, b.

III. TIME INTEGRATION METHOD

A. CENTRAL DIFFERENCE ALGORITHM

Time integration is accomplished using a central difference algorithm. If a_n, b_n, and c_n are known at time $t^{(i)}$, then $\ddot{a}_n^{(i)}$ (the value of $\ddot{a}_n$ at $t^{(i)}$) may be evaluated from the first of equations 14. Letting h represent the time step and $\dot{a}_n^{(i-\frac{1}{2})}$ represent $\dot{a}_n$ at time $t^{(i)} - h/2$, the next value of $\dot{a}_n$ is calculated from

$$\dot{a}_n^{(i+\frac{1}{2})} = \dot{a}_n^{(i-\frac{1}{2})} + h\ddot{a}_n^{(i)} \tag{19}$$

Using this result, the next a_n (at time $t^{(i+1)} = t^{(i)} + h$) is calculated from

$$a_n^{(i+1)} = a_n^{(i)} + h\,\dot{a}_n^{(i+\frac{1}{2})} \tag{20}$$

The value $\ddot{b}_n^{(i)}$ is similarly found from the second of equations 14. A pair of equations paralleling 19 and 20 are used to calculate $\dot{b}_n^{(i+\frac{1}{2})}$ and $b_n^{(i)}$. When these steps have been completed for each n, the value of radial displacement w is found at each structural node from equation 4. These displacements are passed to the fluid program where they furnish required interface boundary conditions to allow an advance to time $t^{(i+1)}$. Values of fluid pressure at the interface nodes are returned by the fluid program.

The new nodal pressures are used to calculate c_n's at $t^{(i+1)}$. It is then again possible to advance $\dot{a}_n$ and a_n using equations 14, 19, and 20 and to perform the parallel calculations for $\dot{b}_n$ and b_n.

The solution process is started with the ring at rest under uniform hydrostatic pressure. Under the loading a_0 is the only nonzero Fourier coefficient. Because $\dot{a}_n^{(0)} = 0$ is known (rather than $\dot{a}_n^{(-\frac{1}{2})}$, a fictitious starting value of $\dot{a}_n^{(-\frac{1}{2})}$ is first calculated from

$$\dot{a}_n^{(-\frac{1}{2})} = -\frac{h}{2}\ddot{a}_n^{(0)}$$

A similar starting procedure is used for $\dot{b}_n$.

B. SELECTION OF THE NUMBER OF VIBRATIONAL MODES USED

The number of vibrational modes that can be modeled accurately is limited by the numerical integration algorithm. Specifically, the algorithm becomes unstable for time steps in excess of about 0.3 of the period of the structural mode. The accuracy of the algorithm deteriorates even before the stability limit is reached. For this reason, a criterion was established for limiting the number of modes used, based on the time steps selected.

In the program the separated equations of motion for the extensional and flexural modes were used to find the frequencies needed in applying the criterion. From

equations 17 and 18 the natural frequencies of the extensional and flexural modes were found to be

$$\omega_{n_{(E)}}^2 = \left(\frac{c}{R}\right)^2 (1 + n^2) \qquad n = 0,1,2,3,\ldots$$

$$\omega_{n_{(F)}}^2 = \left(\frac{c}{R}\right)^2 \left(\frac{r}{R}\right)^2 \frac{n^2(n^2-1)^2}{n^2+1} \qquad n = 2,3,4,\ldots$$

The criterion used was that the period for the highest mode retained be at least five times the time step used. Since the extensional modes have higher frequencies for a given n, this criterion comes into effect first for the extensional modes. Once the point is reached (where $n = n_E(\max)$), the program switches from the coupled equations of motion 14, to the separated equation 17. When this occurs the higher extensional mode coefficients ($n > n_E(\max)$) are not needed, and only the flexural mode coefficients (for $n > n_E(\max)$) are used. If the time step is large enough that the criterion is also met by the flexural modes ($n = n_F(\max)$) then both the a_n's and b_n's are omitted for all modes where n is greater than $n_F(\max)$.

IV. RESULTS

To check the accuracy of the results obtained from the program, two kinds of comparisons were made. First, the solution of equations 14 arrived at by the program were compared to an analytic solution of equations 14. Second, to verify that equations 14 correctly predict ring behavior, calculated dynamic response of the ring to two particular loadings was compared to known static results [Reference 6] for the same loadings.

A. COMPARISON OF MODEL RESULTS TO EXACT SOLUTION

The check on solution accuracy was based on the exact solution for a ring initially at rest and suddenly loaded by a steady pressure

$$P = [1.625 - .25 \cos \theta - .375 \cos 2\theta] \text{ MPa}$$

The exact solution, using the parameters specified in Table I, is

$$\begin{aligned} a_0 &= 4.05844 \times 10^{-3}[1 - \cos (1011.303t)] \\ a_1 &= -159.483t^2 - 1.5617 \times 10^{-4}[1 - \cos (1429.t)] \\ a_2 &= -.104129[1 - \cos (85.616)t] \\ &\quad - 3.7355 \times 10^{-5}[1 - \cos (2259.7t)] \end{aligned} \qquad (22)$$

Table I gives results for radial displacements at $t = 1$ msec for $\theta = 0°$, $90°$ and $180°$. Computer results are given for four different time steps. Even the coarsest time step gives results within 1% of the exact values obtained from equations 22.

B. COMPARISON OF MODEL RESULTS TO STATIC BEHAVIOR

To compare the dynamic response predicted by the program to the known static response of the ring for the same loading, two simple cases of loading were assumed. The first loading case examined was uniform pressure surrounding the ring; the second case was two equal and opposite concentrated loads acting 180° apart as shown in Figure 3. The second loading case could not be represented exactly by the finite trigonometric series loading representation used by the program. It was approximated by nonzero values of pressure only at $\theta = 0°$ and $\theta = 180°$.

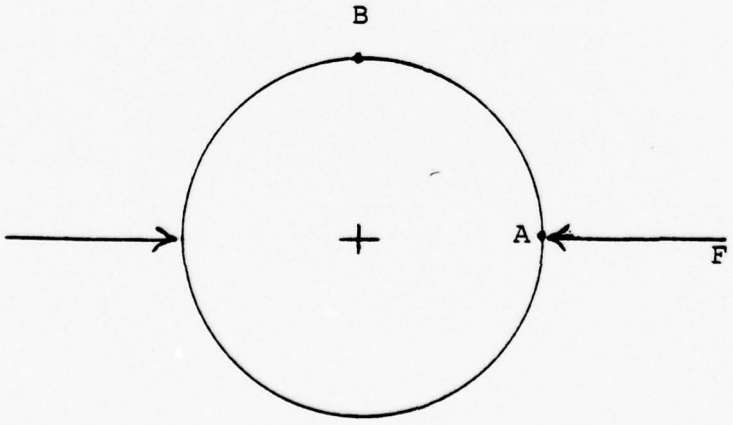

Figure 3. Concentrated Loading on Ring

TABLE I

	Time Step sec	ΔR NP1 $m \times 10^{-7}$	ΔR NP2 $m \times 10^{-7}$	ΔR NP3 $m \times 10^{-7}$
ΔR predicted by program	2.5×10^{-4}	11619.	23687.	17556.
	1×10^{-5}	11679	23472.	17553
	1×10^{-6}	11682	23469.	17555.
	1×10^{-7}	11683.	23469.	17555.
ΔR from equations 17		11683.	23469.	17555.

Ring Parameters: R = 5 m, A = .05 m^2, E = 200 GPa
ρ = 7830 kg/m^3, r = .158m

Comparison of program solution with analytic solution

TABLE II

	ΔR (max) mm	Axial Force (Max) MN
Program results	4.98	-9.8
Twice Known Static Results	5.00	-10.0

Ring Parameters: R = 5 m, A = .05 m^2, E = 200 GPa
ρ = 7830 kg/m^3, r = .158 m

Comparison of program dynamic solution with twice known static solution for uniform external pressure on ring

Comparison between the dynamic and static responses of the ring were made for the radial displacements, bending moments, and forces at particular nodal points on the ring. For the uniform pressure case any nodal point on the ring can be used since the displacements are uniform over the ring. For the concentrated loading case, the nodal points chosen were at the points of application of the forces and midway between the application points.

For the dynamic response of the ring predicted by the program, the ring is taken to be initially at rest and undeformed. The anticipated behavior of the ring after the application of the loading is for the ring to deflect through the radial displacements found for the static loading and to continue to deflect until a radial displacement with a magnitude of approximately twice the static displacement is reached. The maximum deflections were expected to occur after a time approximately equal to one half the period of the vibrational mode that dominates the deflection of the ring. For the uniform loading case the dominant vibrational mode is the fundamental extensional mode. For the concentrated loading case, it is the second flexural mode. The periods for these vibrational modes can be found using equations 21. Tables II and III give the comparisons between the dynamic response from the program and twice the known static response for the two loading cases considered. As is seen, relatively good correlations

TABLE III

Number of Nodal Points Used		ΔR_A (max) m	ΔR_B (max) m	Axial Force at B MN	Bending Moment at B MN·m
5	Model Results	.2965	-.2839	-3.760	-7.910
	Static Analysis Results Times 2	.2634	-.2421	-3.536	-6.434
7	Model Results	.2013	-.1857	-2.353	-4.737
	Static Analysis Results Times 2	.1862	-.1713	-2.500	-4.543
9	Model Results	.1510	-.1393	-1.743	-3.544
	Static Analysis Results Times 2	.1425	-.1311	-1.913	-3.477

Ring Parameters: $R = 5$ m, $A = .05$ m^2, $E = 200$ GPa
$\rho = 7830$ kg/m^3, $r = .158$ m

Comparison of dynamic solution with twice known static solution for loading of Fig. 3

between the dynamic responses and twice the static responses were obtained. The maximum values shown in Table III occur at approximately 35.3 milliseconds after time zero, which compares favorably to one half of the period of the second flexural mode (36.6 milliseconds). The maximum values in Table II occur at 3 milliseconds which is approximately one half the period of the fundamental extensional mode (3.11 milliseconds).

C. OUTPUT EXAMPLES

A sample of the output produced by the structural program is shown in Figures 4 and 5. The print code at the top of Figure 4 is an input code used to select the information desired in the print out. Following the print code are two sets of input parameters to be used in the structural model. These are followed by the mass and stiffness coefficients calculated for each of the vibrational modes. This information remains constant for any given run and is therefore only printed once. The rest of the information varies with time and can be printed as often as desired. The pressures at nodal points generated by the fluid model are printed out; these pressures are for one time step before the time printed below the pressures. Following the time come the Fourier coefficients for the radial displacements, tangential displacements, and pressures. In Figure 5 at the top, the bending moment and the axial force

at each nodal point are printed for the time given in Figure 4. Tabulation of the radial displacement follows.

Figure 6 is an example of the graphical output obtained from the fluid model when the fluid and structural models are run jointly. It is a time sequence of eight plots of fluid nodal pressures over the domain. The plots are shown at 8 ms intervals. The left hand edge of each plot represents the plane of symmetry and the top and bottom rows represent, respectively, the entry and exit faces for the shock. The x's on the lower left side of the plots represent dummy nodes inside the structure. The pressure ranges for the mapping characters are shown in Figure 7. In Figure 6 the development of the cavity can be followed. It develops in the second, third, and fourth frames, collapses in the fifth, and has vanished by the sixth.

```
PRINT CODE: 121111111 1 112 11415161718 1 1

 NNEL  NNP   M    N NNPA1N NNPA2N

 16   17    2    0    9   17

          A         E         R         PI        RHO

        0.500D-01 0.200D 12  5.00     0.158     0.783D 04

    I        XKA1N          XKA2N          XMA1N        XKB1N
    1    0.40040D 09  0.0              391.50       0.0
    2    0.40000D 09  0.10217D 07      391.89       0.40000D 09
    3    0.40360D 09  0.20434D 07      393.07       0.80000D 09
    4    0.42560D 09  0.30651D 07      395.02       0.12000D 10
    5    0.49000D 09  0.40868D 07      397.76       0.16000D 10
    6    0.63040D 09  0.51086D 07      401.29       0.20000D 10
    7    0.89000D 09  0.61303D 07      405.59       0.24000D 10
    8    0.13216D 10  0.71520D 07      410.68       0.28000D 10
    9    0.19876D 10  0.81737D 07      416.56       0.32000D 10
   10    0.29600D 10  0.91954D 07      423.21       0.36000D 10
   11    0.43204D 10  0.10217D 08      430.65       0.40000D 10
   12    0.61600D 10  0.11239D 08      438.87       0.44000D 10
   13    0.85796D 10  0.12261D 08      447.88       0.48000D 10
   14    0.11690D 11  0.13282D 08      457.66       0.52000D 10
   15    0.15610D 11  0.14304D 08      468.23       0.56000D 10
   16    0.20470D 11  0.15326D 08      479.59       0.60000D 10
   17    0.26410D 11  0.16347D 08      491.72       0.64000D 10

 NODAL POINT PRESSURES

0.12D 08  0.98D 07  0.56D 07  0.21D 07  0.52D 06  0.23D 06
0.21D 06  0.20D 06  0.20D 06  0.20D 06  0.21D 06  0.21D 06
0.16D 06  0.20D 06  0.20D 06  0.20D 06  0.20D 06

 TIME =   0.250 MS

      BN              AN              CN
  0.0           0.59923D-03     0.16297D 07
 -0.43636D-05   0.19121D-03     0.27357D 07
 -0.76030D-05   0.16725D-03     0.23797D 07
 -0.90952D-05   0.13425D-03     0.18891D 07
 -0.87962D-05   0.98196D-04     0.13588D 07
 -0.70538D-05   0.63918D-04     0.86480D 06
 -0.48681D-05   0.37398D-04     0.48802D 06
 -0.27813D-05   0.18732D-04     0.23172D 06
 -0.11520D-05   0.69591D-05      76529.
 -0.24765D-06   0.22289D-05      18334.
  0.76762D-08  -0.76762D-07     -8451.9
  0.70517D-07  -0.77569D-06     -15526.
  0.11450D-07  -0.13740D-06     -4825.4
  0.64630D-07  -0.84018D-06     -14242.
  0.48881D-07  -0.68433D-06     -10768.
  0.38325D-07  -0.57487D-06     -8187.1
  0.33831D-07  -0.54130D-06     -9139.7
```

Fig. 4. Output Example

NP	MOMENT	FORCE
1	61466.	-0.22717D 07
2	38662.	-0.21430D 07
3	-31443.	-0.19193D 07
4	-46162.	-0.14334D 07
5	-27598.	-0.11448D 07
6	-7468.4	-0.10173D 07
7	-5081.3	-0.99940D 06
8	-5005.4	-0.10045D 07
9	-5152.0	-0.10006D 07
10	-4923.6	-0.99825D 06
11	-5237.5	-0.10017D 07
12	-3399.1	-0.10010D 07
13	-7444.2	-0.99792D 06
14	-3635.2	-0.99765D 06
15	-5265.8	-0.10009D 07
16	-4963.8	-0.99989D 06
17	-4983.9	-0.99806D 06

NP	NODAL POINT DISPLACEMENTS
1	0.13157D-02
2	0.11784D-02
3	0.86677D-03
4	0.68037D-03
5	0.51822D-03
6	0.50099D-03
7	0.50035D-03
8	0.49977D-03
9	0.49981D-03
10	0.49984D-03
11	0.50032D-03
12	0.50019D-03
13	0.49695D-03
14	0.49939D-03
15	0.49947D-03
16	0.49921D-03
17	0.49944D-03

R; T=10.40/12.73 10.28.52

Fig. 5. Output Example (cont.)

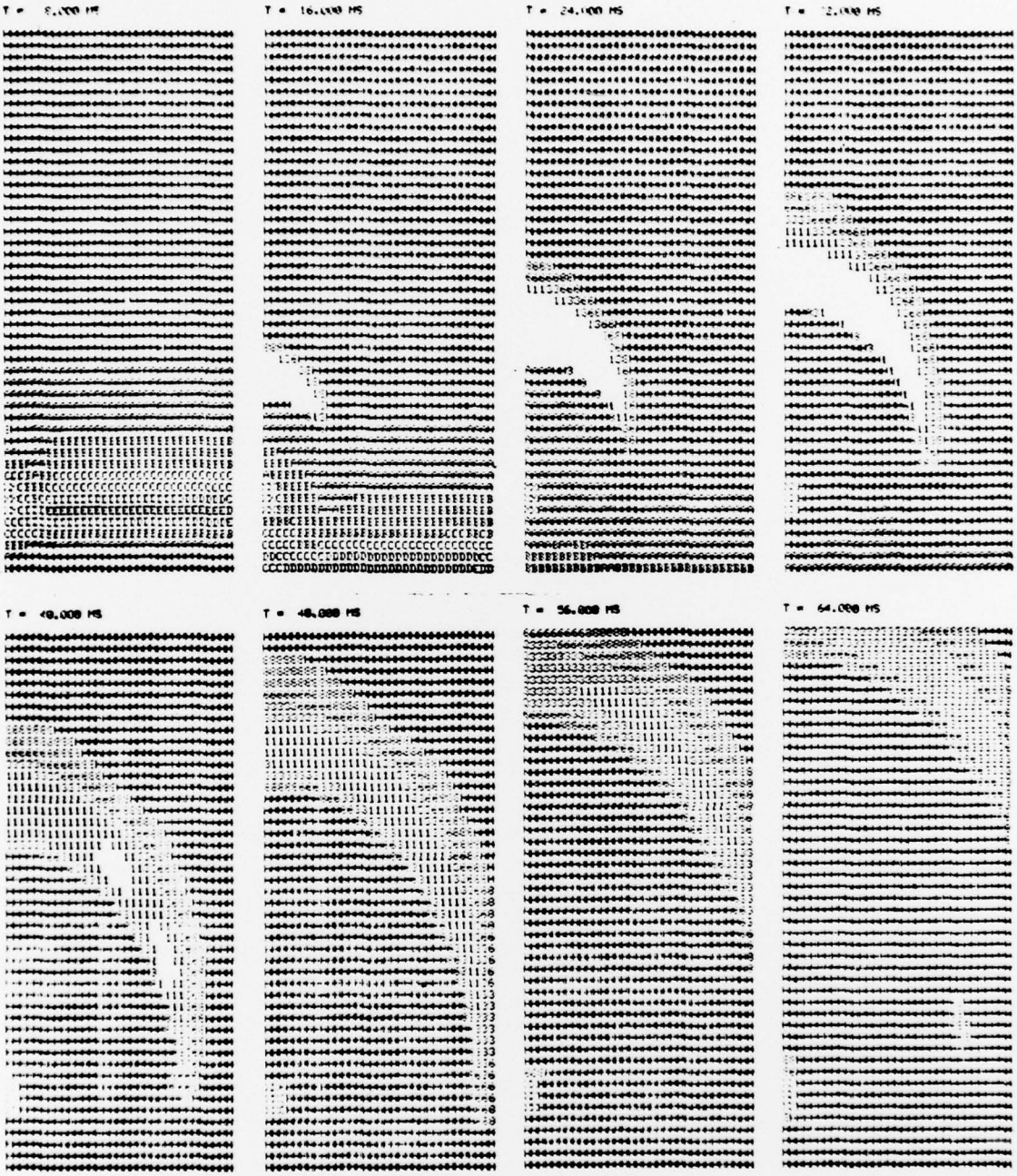

Fig. 6. Cavitation Example

Symbol	Pressure Range (m pa)
X	Structure
blank	Cavitated
1	0 - .04
3	.04 - .09
6	.09 - .14
8	.14 - .18
H	.18 - .43
A	.43 - .77
B	.77 - 1.10
C	1.10 - 1.43
D	1.43 - 1.77
E	1.77 - 2.10
F	over 2.10

Figure 7. Pressure Code

V. CONCLUSIONS AND RECOMMENDATIONS

The comparisons made between the program results where convergence of the solution has been obtained to results obtained by other methods, show that the model satisfactorily predicts the dynamic response of a ring to external pressures. The ring model cannot predict exactly the behavior of a real structure such as a submarine hull. However, by a suitable choice of ring parameters the response of the ring structure will provide a good first approximation to the response of a real structure.

A useful improvement to the program would be to include a section which would automatically pick out maximum values of bending moments and axial forces in the ring as well as when and where they occur. This would allow any desired time intervals between data printout to be used without missing peak responses.

```
C      XP-PRESSURES AT NODAL POINTS AT THE INSTANT OF TIME BEING EXC.      STR00010
C      COSINE-MATRIX OF TERMS, COSINE (NIPI/NNEL)                         STR00020
C      AN-FOURIER COSINE COEFFICIENTS ASSOCIATED WITH RADIAL DISPL.       STR00030
C      BN-FOURIER SINE COEFFICIENT ASSO. WITH TANGENTIAL DISPL.           STR00040
C      ANDOT-1ST TIME DERIVATIVE OF AN                                    STR00050
C      ANDDT-SECOND TIME DERIVATIVE OF AN                                 STR00060
C      BNDOT-1ST TIME DERIVATIVE OF BN                                    STR00070
C      BNDDT-2ND TIME DERIVATIVE OF BN                                    STR00080
C      NNEL-NUMBER OF ELEMENTS                                            STR00090
C      NNP-NUMBER OF NODAL POINTS                                         STR00100
C      XK1N-STIFFNESS COEFFICIENTS FOR FLEXURAL MODE                      STR00110
C      XM1N-MASS COEFFICIENTS FOR FLEXURAL MODE                           STR00120
C      XKA1N-STIFFNESS COEFF. ASSO. WITH RADIAL DISPL. USED IN            STR00130
C     *CALCULATING ANDDT                                                  STR00140
C      XKB1N-STIFFNESS COEFF. ASSO. WITH TANGENTIAL DISPL. USED IN        STR00150
C     *CALCULATING ANDDT                                                  STR00160
C      XMA1N-MASS COEFF. ASSO. WITH RADIAL DISPLACEMENT                   STR00170
C      XKA2N-STIFFNESS COEFF. ASSO. WITH RADIAL DISPL. USED IN            STR00180
C     *CALCULATING BNDDT                                                  STR00190
C      XKB2N-STIFFNESS COEFF. ASSO. WITH TANGENTIAL DISPL. USED IN        STR00200
C     *CALCULATING BNDDT                                                  STR00210
C      NNPA1N-MAXIMUM NUMBER OF MODES FOR WHICH AN AND BN ARE SOLVED      STR00220
C     *FOR SIMULTANEOUSLY                                                 STR00230
C      NNPA2N-MAXIMUM NO. OF MODES INCLUDED IN CALCULATIONS               STR00240
C      K,L,LLL,LLLL-COUNTERS                                              STR00250
C      H-TIME INCREMENT                                                   STR00260
C      RHO-DENSITY OF RING MATERIAL                                       STR00270
C      AA-CROSS SECTIONAL AREA                                            STR00280
C      RI-RADIUS OF GYRATION                                              STR00290
C      R-RADIUS OF RING                                                   STR00300
C      E-MATERIAL MODULUS OF ELASTICITY                                   STR00310
C      MM-NUMBER OF LOOPS DESIRED BETWEEN PRINTOUTS                       STR00320
C      ZN-NORMAL FORCE IN RING                                            STR00330
C      ZM-BENDING MOMENT                                                  STR00340
C      W-RADIAL DISPLACEMENT                                              STR00350
C      PH-HYDROSTATIC PRESSURE                                            STR00360
C      ZPI-PI                                                             STR00370
C      THETA-ANGLE BETWEEN NORMAL TO SHOCK FRONT AND LOCATION ON RING     STR00380
C                                                                         STR00390
C                                                                         STR00400
C                                                                         STR00410
C                                                                         STR00420
      SUBROUTINE INPUT                                                    STR00430
C                                                                         STR00440
C                                                                         STR00450
      IMPLICIT REAL*8 (A-H,O-Z)                                           STR00460
      COMMON/OUT/K(20)                                                    STR00470
      COMMON/SINT/NNEL,NNP,MM,NN,NNPA1N,NNPA2N,L,LLL                      STR00480
```

```
      COMMON/REAL/HH,AA,E,R,RI,RHO,GC,TI                                STR00490
      COMMON/VECTS/XP(50),COSINE(50,50),ANDDT(50),BN(50)                STR00500
     *,BNDDT(50),ANDOT(50),BNDOT(50),XKA1N(50),XMA1N(50),AN(50)         STR00510
     *,XKA2N(50),XKB1N(50),XKB2N(50),CN(50),XM1N(50),XK1N(50)           STR00520
      COMMON/IPASS/DT,A,PH,N                                            STR00530
      COMMON/SPASS/PASS(50)                                             STR00540
                                                                        STR00550
C       READ NEEDED CONSTANTS                                           STR00560
                                                                        STR00570
      ZPI=4.DO*DATAN(1.DO)                                              STR00580
      NNP=4*N+1                                                         STR00590
      NNEL=NNP-1                                                        STR00600
      HH=DT                                                             STR00610
      R=A                                                               STR00620
      L=0                                                               STR00630
      READ(5,26)MM                                                      STR00640
      READ(5,27)AA,E,RI,RHO                                             STR00650
26    FORMAT(615)                                                       STR00660
27    FORMAT(7G10.3)                                                    STR00670
      EXTN=2*ZPI*R/((5*DT)*(E/RHO)**.5)                                 STR00680
      FLEXN=(EXTN*R/RI)**.5                                             STR00690
      NNPA1N=EXTN                                                       STR00700
      NNPA2N=FLEXN                                                      STR00710
      IF(NNP.LT.NNPA1N)NNPA1N=NNP                                       STR00720
32    IF(NNP.LT.NNPA2N)NNPA2N=NNP                                       STR00730
      RETURN                                                            STR00740
      END                                                               STR00750
                                                                        STR00760
                                                                        STR00770
      SUBROUTINE COEFFI                                                 STR00780
                                                                        STR00790
                                                                        STR00800
      IMPLICIT REAL*8(A-H,O-Z)                                          STR00810
      COMMON/SINT/NNEL,NNP,MM,NN,NNPA1N,NNPA2N,L,LLL                    STR00820
      COMMON/REAL/HH,AA,E,R,RI,RHO,GC,TI                                STR00830
      COMMON/VECTS/XP(50),COSINE(50,50),ANDDT(50),BN(50)                STR00840
     *,BNDDT(50),ANDOT(50),BNDOT(50),XKA1N(50),XMA1N(50),AN(50)         STR00850
     *,XKA2N(50),XKB1N(50),XKB2N(50),CN(50),XM1N(50),XK1N(50)           STR00860
      COMMON/IPASS/DT,A,PH,N                                            STR00870
                                                                        STR00880
      RIR=RI/R                                                          STR00890
      EAORS=E*AA/(R*R)                                                  STR00900
      RA=RHO*AA                                                         STR00910
      ECRHRS=E/(RHO*R**2)                                               STR00920
      Z=RIR**2                                                          STR00930
      ZPI=4.DO*DATAN(1.DO)                                              STR00940
                                                                        STR00950
C     FORM FOURIER COEFFICIENTS FOR PRESSURE REPRESENTATION AND         STR00960
```

```
C       FORM MASS AND STIFFNESS COEFFICIENTS                                    STR00970
                                                                                STR00980
        DO 500 I=1,NNP                                                          STR00990
        XI=I                                                                    STR01000
        DO 505 J=1,NNP                                                          STR01010
        XJ=J                                                                    STR01020
        XNEL=NNEL                                                               STR01030
        THETA=(XI-1.DO)*(XJ-1.DO)*ZPI/XNEL                                      STR01040
        COSINE(I,J)=DCOS(THETA)                                                 STR01050
505     CONTINUE                                                                STR01060
        XI=XI-1.DO                                                              STR01070
        XMAIN(I)=RA*(1.DO+Z*XI**2)                                              STR01080
        XKAIN(I)=EAORS*(Z*(XI**2-1.DO)**2+1.DO)                                 STR01090
        XKB1N(I)=EACRS*XI                                                       STR01100
        XKA2N(I)=EORHRS*XI                                                      STR01110
        XKB2N(I)=EORHRS*XI**2                                                   STR01120
        IF(I.EQ.1)GO TO 582                                                     STR01130
        XM1N(I)=RA*((XI**2+1.DO)/XI**2+Z*XI**2)                                 STR01140
582     XK1N(I)=EAORS*(RIR**2*(XI**2-1.DO)**2)                                  STR01150
500     CONTINUE                                                                STR01160
        CALL PRINT2(14)                                                         STR01170
                                                                                STR01180
C       SET COEFFICIENTS = 0 INITALLY                                           STR01190
                                                                                STR01200
        PH=PH/XKAIN(1)                                                          STR01210
        DO 526 I=1,NNP                                                          STR01220
        AN(I)=0.DO                                                              STR01230
        ANDOT(I)=0.DO                                                           STR01240
        BNDOT(I)=0.DO                                                           STR01250
526     BN(I)=0.DO                                                              STR01260
        AN(1)=PH                                                                STR01270
        LLL=1                                                                   STR01280
        TI=0.DO                                                                 STR01290
        L=1                                                                     STR01300
        RETURN                                                                  STR01310
        END                                                                     STR01320
                                                                                STR01330
        SUBROUTINE TIMSTP                                                       STR01340
                                                                                STR01350
        IMPLICIT REAL*8(A-H,O-Z)                                                STR01360
        COMMON/SINT/NNEL,NNP,MM,NN,NNPA1N,NNPA2N,L,LLL                          STR01370
        COMMON/REAL/HH,AA,E,R,RI,RHO,GC,TI                                      STR01380
                                                                                STR01390
        CALL PINPUT                                                             STR01400
        CALL CNS                                                                STR01410
        TI=LLL*HH                                                               STR01420
        CALL PREDIC                                                             STR01430
        CALL MOMFOR                                                             STR01440
```

```
      CALL DISPLA                                                       STR01450
      L=L+1                                                             STR01460
      LLL=LLL+1                                                         STR01470
      RETURN                                                            STR01480
76    STOP                                                              STR01490
      END                                                               STR01500
                                                                        STR01510
                                                                        STR01520
      SUBROUTINE PINPUT                                                 STR01530
                                                                        STR01540
                                                                        STR01550
      IMPLICIT REAL*8(A-H,O-Z)                                          STR01560
      COMMON/SINT/NNEL,NNP,MM,NN,NNPA1N,NNPA2N,L,LLL                    STR01570
      COMMON/REAL/HH,AA,E,R,RI,RHO,GC,TI                                STR01580
      COMMON/VECTS/XP(50),COSINE(50,50),ANDOT(50),BN(50)                STR01590
     *,BNDOT(50),ANDOT(50),BNDOT(50),XKA1N(50),XMA1N(50),AN(50)         STR01600
     *,XKA2N(50),XKB1N(50),XKB2N(50),CN(50),XM1N(50),XK1N(50)           STR01610
      COMMON/SPASS/PASS(50)                                             STR01620
5     FORMAT(6G10.2)                                                    STR01630
      DO 6 I=1,NNP                                                      STR01640
6     XP(I)=PASS(I)                                                     STR01650
      XP(1)=XP(1)/2.DO                                                  STR01660
      XP(NNP)=XP(NNP)/2.DO                                              STR01670
      CALL PRINT2(15)                                                   STR01680
      RETURN                                                            STR01690
      END                                                               STR01700
                                                                        STR01710
                                                                        STR01720
      SUEROUTINE CNS                                                    STR01730
                                                                        STR01740
                                                                        STR01750
      IMPLICIT REAL*8(A-H,O-Z)                                          STR01760
      COMMON/SINT/NNEL,NNP,MM,NN,NNPA1N,NNPA2N,L,LLL                    STR01770
      COMMON/REAL/HH,AA,E,R,RI,RHO,GC,TI                                STR01780
      COMMON/VECTS/XP(50),COSINE(50,50),ANDOT(50),BN(50)                STR01790
     *,BNDOT(50),ANDOT(50),BNDOT(50),XKA1N(50),XMA1N(50),AN(50)         STR01800
     *,XKA2N(50),XKB1N(50),XKB2N(50),CN(50),XM1N(50),XK1N(50)           STR01810
                                                                        STR01820
C       SOLVE FOR CN'S                                                  STR01830
                                                                        STR01840
      DO 60 I=1,NNP                                                     STR01850
      C=0.DO                                                            STR01860
      DO 65 J=1,NNP                                                     STR01870
      C=XP(J)*COSINE(I,J)+C                                             STR01880
65    CONTINUE                                                          STR01890
      XNEL=NNEL                                                         STR01900
      CN(I)=(2.DO/XNEL)*C                                               STR01910
      IF(I.EQ.1)CN(I)=CN(I)/2.DO                                        STR01920
```

```
      IF(I.EQ.NNP)CN(I)=CN(I)/2.DO                                          STR01930
60    CONTINUE                                                              STR01940
      RETURN                                                                STR01950
       END                                                                  STR01960
                                                                            STR01970
                                                                            STR01980
      SUBROUTINE PREDIC                                                     STR01990
                                                                            STR02000
                                                                            STR02010
      IMPLICIT REAL*8(A-H,O-Z)                                              STR02020
      COMMON/SINT/NNEL,NNP,MM,NN,NNPA1N,NNPA2N,L,LLL                        STR02030
      COMMON/REAL/HH,AA,E,R,RI,RHO,GC,TI                                    STR02040
      COMMON/VECTS/XP(50),COSINE(50,50),ANDDT(50),BN(50)                    STR02050
     *,BNDDT(50),ANDOT(50),BNDOT(50),XKA1N(50),XMA1N(50),AN(50)             STR02060
     *,XKA2N(50),XKB1N(50),XKB2N(50),CN(50),XM1N(50),XK1N(50)               STR02070
                                                                            STR02080
C     CENTRAL DIFFERENCE INTEGRATION OF D.E. TO PREDICT FUTURE A1N"S        STR02090
                                                                            STR02100
      IF(LLL.NE.1)GO TO 541                                                 STR02110
      DO 542 I=1,NNP                                                        STR02120
      IF(I.GT.NNPA2N)GO TO 542                                              STR02130
      IF(I.GT.NNPA1N)GO TO 543                                              STR02140
      ANDDT(I)=(CN(I)-XKA1N(I)*AN(I)-XKB1N(I)*BN(I))/XMA1N(I)               STR02150
      ANDOT(I)=-.5DO*ANDDT(I)*HH                                            STR02160
      BNDDT(I)=-(XKA2N(I)*AN(I)+XKB2N(I)*BN(I))                             STR02170
      BNDOT(I)=-.5DO*BNDDT(I)*HH                                            STR02180
      GO TO 542                                                             STR02190
543   ANDDT(I)=(CN(I)-AN(I)*XK1N(I))/XM1N(I)                                STR02200
      ANDOT(I)=-.5DO*ANDDT(I)*HH                                            STR02210
542   CONTINUE                                                              STR02220
541   DO 540 I=1,NNP                                                        STR02230
      IF(I.GT.NNPA1N)GO TO 550                                              STR02240
      XI=I                                                                  STR02250
      ANDDT(I)=(CN(I)-XKA1N(I)*AN(I)-XKB1N(I)*BN(I))/XMA1N(I)               STR02260
      ANDOT(I)=ANDOT(I)+ANDDT(I)*HH                                         STR02270
      AN(I)=AN(I)+ANDOT(I)*HH                                               STR02280
      BNDDT(I)=-(XKA2N(I)*AN(I)+XKB2N(I)*BN(I))                             STR02290
      BNDOT(I)=BNDOT(I)+BNDDT(I)*HH                                         STR02300
      BN(I)=BN(I)+BNDOT(I)*HH                                               STR02310
      GO TO 540                                                             STR02320
550   CONTINUE                                                              STR02330
      IF(I.GT.NNPA2N)GO TO 540                                              STR02340
      ANDDT(I)=(CN(I)-AN(I)*XK1N(I))/XM1N(I)                                STR02350
      ANDOT(I)=ANDOT(I)+HH*ANDDT(I)                                         STR02360
      AN(I)=AN(I)+HH*ANDOT(I)                                               STR02370
      XI=I-1                                                                STR02380
      BN(I)=-AN(I)/XI                                                       STR02390
540   CONTINUE                                                              STR02400
```

```
      CALL PRINT2(17)                                                    STR02410
      RETURN                                                             STR02420
      END                                                                STR02430
                                                                         STR02440
                                                                         STR02450
      SUBROUTINE MOMFOR                                                  STR02460
                                                                         STR02470
                                                                         STR02480
      IMPLICIT REAL*8(A-H,O-Z)                                           STR02490
      COMMON/DIS/W(50)                                                   STR02500
      COMMON/SINT/NNEL,NNP,MM,NN,NNPA1N,NNPA2N,L,LLL,LLLL,J              STR02510
      COMMON/REAL/HH,AA,E,R,RI,RHO,GC,TI,ZN,ZM                           STR02520
      COMMON/VECTS/XP(50),COSINE(50,50),ANDDT(50),BN(50)                 STR02530
     *,BNDDT(50),ANDOT(50),BNDOT(50),XKA1N(50),XMA1N(50),AN(50)          STR02540
     *,XKA2N(50),XKB1N(50),XKB2N(50),CN(50),XMIN(50),XK1N(50)            STR0255C
                                                                         STR02560
C     CALCULATE BENDING MOMEMTS AND AXIAL FORCE                          STR02570
                                                                         STR02580
      ZPI=4.DC*DATAN(1.DO)                                               STR02590
      Z=(RI/R)**2                                                        STR02600
      LLLL=1                                                             STR02610
      C=-E*AA*RI**2/R**2                                                 STR02620
      B=E*AA/R                                                           STR02630
      DO 601 J=1,NNP                                                     STR02640
      ZM=0.DO                                                            STR02650
      ZN=0.DO                                                            STR02660
      DO 602 I=1,NNP                                                     STR02670
      XI=I-1                                                             STR02680
      XJ=J-1                                                             STR02690
      ZM=C*(1-XI**2)*AN(I)*DCOS(XI*XJ*ZPI/NNEL)+ZM                       STR02700
602   ZN=-B*(AN(I)*(1+Z*(1-XI**2))+XI*BN(I))*DCOS(XI*XJ*ZPI/NNEL)+ZN     STR02710
601   CALL PRINT2(16)                                                    STR02720
      LLLL=1                                                             STR02730
      RETURN                                                             STR02740
      END                                                                STR02750
                                                                         STR02760
                                                                         STR0277C
      SUBROUTINE DISPLA                                                  STR02780
                                                                         STR02790
                                                                         STR02800
      IMPLICIT REAL*8(A-H,O-Z)                                           STR02810
      COMMON/DIS/W(50)                                                   STR02820
      COMMON/SINT/NNEL,NNP,MM,NN,NNPA1N,NNPA2N,L,LLL                     STR0283C
      COMMON/REAL/HH,AA,E,R,RI,RHO,GC,TI                                 STR02840
      COMMON/VECTS/XP(50),COSINE(50,50),ANDDT(50),BN(50)                 STR02850
     *,BNDDT(50),ANDOT(50),BNDOT(50),XKA1N(50),XMA1N(50),AN(50)          STR02860
     *,XKA2N(50),XKB1N(50),XKB2N(50),CN(50),XMIN(50),XK1N(50)            STR02870
      COMMON/SPASS/PASS(50)                                              STR0288C
```

```
C       FIND RADIAL DISPLACEMENT                                                STR02890
                                                                                STR02900
                                                                                STR02910
        DO 570 I=1,NNP                                                          STR02920
        WT=0.DO                                                                 STR02930
        DO 575 J=1,NNP                                                          STR02940
        V=AN(J)*COSINE(I,J)                                                     STR02950
        WT=WT+V                                                                 STR02960
575     CONTINUE                                                                STR02970
        W(I)=WT                                                                 STR02980
        PASS(I)=W(I)                                                            STR02990
570     CONTINUE                                                                STR03000
        CALL PRINT2(18)                                                         STR03010
        RETURN                                                                  STR03020
        END                                                                     STR03030
                                                                                STR03040
                                                                                STR03050
        SUBROUTINE PRINT2(LL)                                                   STR03060
                                                                                STR03070
                                                                                STR03080
        IMPLICIT REAL*8(A-H,O-Z)                                                STR03090
        COMMON/DIS/W(50)                                                        STR03100
        COMMON/OUT/K(20)                                                        STR03110
        COMMON/SINT/NNEL,NNP,MM,NN,NNPA1N,NNPA2N,L,LLL,LLLL,J                   STR03120
        COMMON/REAL/HH,AA,E,R,RI,RHO,GC,TI,ZN,ZM                                STR03130
        COMMON/VECTS/XP(50),COSINE(50,50),ANDDT(50),BN(50)                      STR03140
     *,ENDDT(50),ANDOT(50),BNDOT(50),XKA1N(50),XMA1N(50),AN(50)                 STR03150
     *,XKA2N(50),XKB1N(50),XKB2N(50),CN(50),XM1N(50),XK1N(50)                   STR03160
                                                                                STR03170
        IF(L.EQ.0)GO TO 91                                                      STR03180
        IF(L.EQ.MM)GO TO 91                                                     STR03190
        GO TO 1                                                                 STR03200
91      IF(K(LL).EQ.1) GO TO 1                                                  STR03210
        GO TO (1,1,1,1,1,1,1,1,1,1,1,1,1,14,15,16,17,18),LL                    STR03220
1       RETURN                                                                  STR03230
                                                                                STR03240
C       PRINT INPUT DATA & MASS AND STIFFNESS COEFFICIENTS                      STR03250
                                                                                STR03260
                                                                                STR03270
14      WRITE(6,978)                                                            STR03280
978     FORMAT(/3X,'STRUK5',/)                                                  STR03290
        WRITE(6,85)                                                             STR03300
85      FORMAT(/3X,'NNEL  NNP  MM NNPA1N NNPA2N',/)                             STR03310
27      FORMAT(10X,5G10.3)                                                      STR03320
        WRITE(6,26)NNEL,NNP,MM,NNPA1N,NNPA2N                                    STR03330
        WRITE(6,90)                                                             STR03340
90      FORMAT(/,13X,'A',5X,'E',9X,'R',9X,'RI',7X,'RHO',/)                      STR03350
        WRITE(6,27)AA,E,R,RI,RHO                                                STR03360
```

```
26    FORMAT(6I5)                                                          STR03370
      WRITE(6,1010)                                                        STR03380
1010  FORMAT(/,6X,'I',8X,'XKA1N',8X,'XKA2N',8X,'XMA1N',8X,'XKB1N')         STR03390
      WRITE(6,1015)(I,XKA1N(I),XKA2N(I),XMA1N(I),XKB1N(I),                 STR03400
     *I=1,NNP)                                                             STR03410
1015  FORMAT(I8,2X,4G13.5)                                                 STR03420
      RETURN                                                               STR03430
                                                                           STR03440
                                                                           STR03450
C     PRINT INPUT PRESSURE                                                 STR03460
                                                                           STR03470
                                                                           STR03480
15    WRITE(6,75)                                                          STR03490
      XP(1)=XP(1)*2.D0                                                     STR03500
      XP(NNP)=XP(NNP)*2.D0                                                 STR03510
75    FORMAT(/,3X,'NODAL POINT PRESSURES',/)                               STR03520
      WRITE(6,5)(XP(I),I=1,NNP)                                            STR03530
      XP(1)=XP(1)/2.D0                                                     STR03540
      XP(NNP)=XP(NNP)/2.D0                                                 STR03550
5     FORMAT(6G10.2)                                                       STR03560
      RETURN                                                               STR03570
                                                                           STR03580
                                                                           STR03590
C     PRINT BENDING MOMENTS & AXIAL FORCES                                 STR03600
                                                                           STR03610
                                                                           STR03620
16    IF(LLLL.NE.1)GO TO 606                                               STR03630
      LLLL=LLLL+1                                                          STR03640
      WRITE(6,604)                                                         STR03650
604   FORMAT(/,'      NP                MOMENT                FORCE',/)    STR03660
606   WRITE(6,603)J,ZM,ZN                                                  STR03670
603   FORMAT(3X,I5,8X,G15.5,8X,G15.5)                                      STR03680
      RETURN                                                               STR03690
                                                                           STR03700
                                                                           STR03710
C     PRINT TIME, AN,BN, & CN                                              STR03720
                                                                           STR03730
                                                                           STR03740
17    WRITE(6,70) TI                                                       STR03750
70    FORMAT(/,3X,'TIME =',3PF8.3,' MS')                                   STR03760
      WRITE(6,1020)                                                        STR03770
1020  FORMAT(/,2X,'      BN                AN                CN')         STR03780
      WRITE(6,1000)(BN(I),AN(I),CN(I),I=1,NNP)                             STR03790
1000  FORMAT(3G15.5)                                                       STR03800
      RETURN                                                               STR03810
                                                                           STR03820
                                                                           STR03830
C     PRINT NODAL POINT DISPLACEMENTS                                      STR03840
```

```
18      WRITE(6,80)                                                        STR03850
        WRITE(6,35)(I,W(I),I=1,NNP)                                        STR03860
35      FORMAT(I5,12X,G15.5)                                               STR03870
80      FORMAT(/,'    NP    NODAL POINT DISPLACEMENTS',/)                  STR03880
        L=0                                                                STR03890
        RETURN                                                             STR03900
        END                                                                STR03910
                                                                           STR03920
                                                                           STR03930
```

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